Cavity Radius vs Energy Dissipation Rate in Liquid-Filled, Precessing, Spherical Cavities

JAMES P. VANYO*
University of California, Santa Barbara, Calif.

Theme

EXPERIMENTAL results and an idealized model are presented that yield energy dissipation rates as a function of radius for liquid-filled, precessing, spherical cavities. Experiments were conducted using cavities 11, 16.5, and 22 cm in diameter and filled first with water to give turbulent flow conditions, and next with a silicone liquid ($\nu = 1000$ cs) to give laminar flow conditions. The objective of the experiments was to determine whether energy dissipation would be minimized, for a given mass of liquid, by using many small cavities rather than one, or a few, large cavities. An affirmative result would indicate that the use of baffles in a large cavity might, in the same manner, reduce energy dissipation.

Contents

Use of the energy sink criteria for assessing attitude stability of a spinning and precessing satellite has led to an interest in energy dissipation rates within liquids (e.g., fuels) carried aboard the satellite. A number of researchers have studied the problem both experimentally and analytically. A survey of prior work is given in the references.

Vanyo and Likins presented results[1] for energy dissipation rates in a liquid-filled, precessing cavity with a diameter of 22 cm using water and a 20-cs silicone liquid. They also presented a model[2] for energy dissipation rate (P) that treated the net motion of the liquid as a rigid sphere separated from the cavity wall by an Ekman-type boundary layer. An equation was derived as

$$P = [4\pi(2)^{1/2}\rho/3(1+\zeta^2)]R^4\nu^{1/2}\dot{\psi}^2(\dot{\phi}+\dot{\psi}\cos\theta)^{1/2}\sin^2\theta \quad (1)$$

where ρ is density, R is the radius, ν is kinematic viscosity, $\dot{\phi}$ is precession speed (relative to an inertial frame), $\dot{\psi}$ is spin speed (relative to the precessing frame), and θ is the half coning angle. The dimensionless parameter ζ is related to an Ekman number and is discussed later.

Figure 1 of Ref. 1 shows the apparatus which rotates a test cavity about the spin axis ($\dot{\psi}$) while the spin axis precesses ($\dot{\phi}$) at a fixed coning angle θ. Figure 3 of Ref. 1 shows a summary of the prior experimental results. Energy dissipation is computed as

$$P = \dot{\psi}_M \times \dot{\psi} \quad (2)$$

where $\dot{\psi}_M$ is the measured component of moment in the direction of the spin axis. Significant among the prior results were the differences in energy dissipation between prolate and oblate type precession and the rapid increase in energy dissipation at very small $\dot{\phi}$ (1–3 rpm) when using water as the test liquid, especially for the prolate case. It was shown that $\dot{\psi}_M$ (and P) reach steady-state values independent of $\dot{\phi}$ over the range of approximately $10 < \dot{\phi} < 150$ rpm. This is the region of "saturated" turbulence. In this region, the prolate and oblate results converge. A later paper[3] examined the region of very small $\theta (< 1°)$ and $\dot{\psi} (< 10$ rpm) including use of a 1000 cs silicone liquid. That research demonstrated a slow increase (nearly linear) of $\dot{\psi}_M$ and P as $\dot{\phi}$ is increased. A dimensionless presentation showed the flow parameters in that regime to be associated with slow laminar flow.

The results presented here were obtained with the same apparatus and instrumentation used in Refs. 1 and 3 except that the 22-cm-diam tank was replaced with tanks having 16.5 cm diam and 11 cm diam, respectively. In each case water ($\nu = 1$ cs) was used to measure turbulent flows and the 1000-cs silicone liquid was used to measure laminar flows. The test parameters otherwise corresponded to like parameters of the 22-cm-diam cavity tests so that the dependence on R could be analyzed.

Figure 1 shows typical results for the three cavity sizes for the laminar and near laminar region. A large set of data, in the same form as that shown in Fig. 1, was obtained for both turbulent and laminar flows. Comparison of the results for identical parameters ρ, ν, θ, $\dot{\phi}$, and $\dot{\psi}$ yields relationships $P = P(R)$ or equivalently $\dot{\psi}_M = \dot{\psi}_M(D)$, where D is diameter.

Figure 2 shows a comparison of $\dot{\psi}_M$ results for the three tank sizes with water in turbulent flow ($\dot{\phi} = 40$ rpm). The data shows $\dot{\psi}_M = \dot{\psi}_M(D)$ over a series of tests with parameters given in the format ($\dot{\psi}, \theta$) at the upper point of typical lines. The solid lines correspond to results for prolate type precession ($\dot{\phi}\dot{\psi} > 0$) and the dashed lines to results for oblate type precession

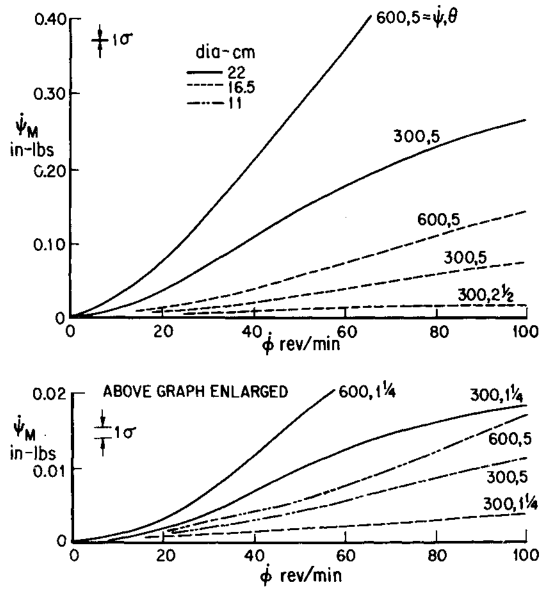

Fig. 1 Spin motor moment ($\dot{\psi}_M$) vs precession speed ($\dot{\phi}$). Silicone liquid ($\nu = 1000$ cs). D, $\dot{\psi}$, and θ as shown.

Received July 17, 1972; synoptic received October 26, 1972. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N73-10319 at the standard price (available upon request).

Index categories: Spacecraft Attitude Dynamics and Control; Wave Motion and Sloshing.

* Assistant Professor, Department of Mechanical Engineering.

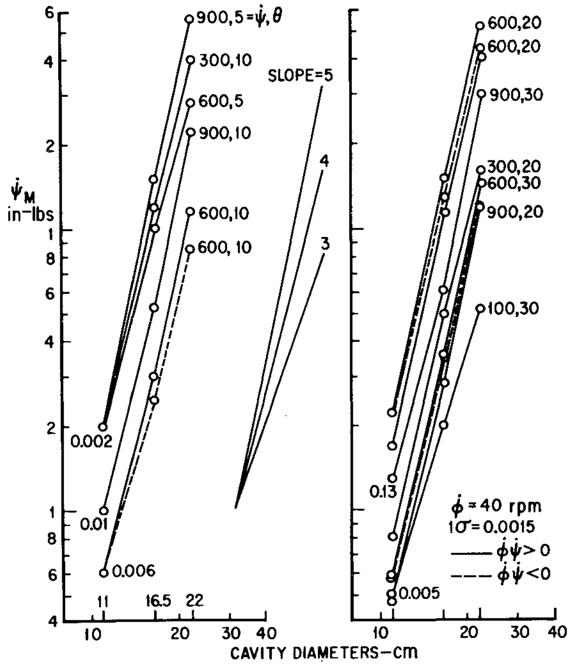

Fig. 2 Spin motor moment vs cavity diameter (D). Water ($\nu = 1$ cs); $\dot\psi$, θ, $\dot\phi$ as shown. All points represent turbulent flow conditions.

($\dot\phi\dot\psi < 0$). The results at the left of the graph are for $\theta = 5°$ and 10° and at the right for $\theta = 20°$ and 30°.

The numbers at the lower point of the lines identify the appropriate scales for ordinates of typical lines. Although all the results are shown on parts of 3 logarithmic cycles, they actually span parts of 4 cycles (from 0.002 to 3.0 in.-lb). Inclusion of the factor $\dot\psi$ leads to a span of 5 cycles for P. Therefore the results are representative of a wide range of energy dissipation rates. The mean slope of the results for $\theta = 5°$ and 10° is approximately 4.10, and for $\theta = 20°$ and 30° it is approximately 4.55 leading to

$$P(\text{turbulent}) \propto R^{4.55} \qquad 20° \leq \theta \leq 30° \tag{3a}$$

$$P(\text{turbulent}) \propto R^{4.10} \qquad 5° \leq \theta \leq 10° \tag{3b}$$

Results for $\theta < 5°$ could not be obtained because the values of $\dot\psi_M$ for the 11-cm-diam tank (using water) were generally less than the 1σ value of $\dot\psi_M \approx 0.0015$ in.-lb.

Results were also obtained for laminar flow using the 1000 cs silicone liquid. In laminar flow there is no "flat" region corresponding to saturated turbulence, so results were obtained for three values of $\dot\phi$ of 30, 60, and 90 rpm. The functional form $P = P(R)$ did not vary with changes in $\dot\phi$ over the region $30 \leq \dot\phi \leq 90$ rpm. The mean slope of the laminar flow results for $\theta = 5°$ is 5.25. For $\theta = 2\frac{1}{2}°$ and $1\frac{1}{4}°$, the most significant results indicate a slope on the order of 5.45 leading to

$$P(\text{laminar}) \propto R^{5.25} \qquad \theta = 5° \tag{4a}$$

$$P(\text{laminar}) \propto R^{5.45} \qquad 1\tfrac{1}{4}° \leq \theta \leq 2\tfrac{1}{2}° \tag{4b}$$

The dimensionless parameter ζ used in Eq. (1) is derived in Ref. 2 to be

$$\zeta = 5\nu/hR\dot\phi \tag{5a}$$

where h is the separation distance between the rigid interior sphere and the cavity wall. Its magnitude is related to an Ekman layer thickness and is given as

$$h = (2)^{1/2}[\nu/(\dot\phi + \dot\psi\cos\theta)]^{1/2} \tag{5b}$$

Experimental results confirm that $\zeta \leq 1$ corresponds to fully turbulent flow in a spherical cavity and $\zeta \geq 80$ corresponds to slow laminar flow. The region from $\zeta = 80$ to $\zeta = 1$ is associated with wave phenomena and the growth of turbulence.

With θ small, $\dot\phi$ small relative to $\dot\psi$, and $\zeta^2 \ll 1$, the form of Eq. (1) reduces to

$$P = [4\pi(2)^{1/2}/3]\rho R^4 \nu^{1/2}\theta^2\dot\psi^{2.5} \tag{6}$$

In laminar flow, the approximations θ small and $\zeta^2 \gg 1$ leads to

$$P = [8\pi(2)^{1/2}/75]\rho R^6 \nu^{-1/2}\theta^2\dot\psi^2\dot\phi^2(\dot\phi + \dot\psi)^{-1/2} \tag{7}$$

In the laminar flow experiments, $\dot\psi$ was often on the order of $\dot\phi$, and the assumption $\dot\phi \ll \dot\psi$ is not appropriate.

The experimental results presented here showing $P \propto R^{4.55}$ for large θ and $P \propto R^{4.10}$ for $\theta = 5°$ are in general agreement with the form R^4 in Eq. (6). Although not experimentally verified, it may be that R^4 is the limiting case for very small θ. The rigid interior sphere model was originally derived for the case of fully turbulent flow. Reference 3, however, shows empirical equations for the case of laminar flow which in some parameter regions compare favorably with Eq. (7). The experimental verification here of $P \propto R^{5.25}$ to $R^{5.45}$ also shows a dependence similar to the R^6 dependence in Eq. (7).

In communication satellite attitude stability studies, it is often desirable to minimize energy dissipation for a given mass of liquid fuel. Here the dependence of P on R may be critical, and a design option of n smaller cavities rather than 1 large cavity is often available. If $P \propto R^3$, there is no advantage in using either larger or smaller cavities. However, if $P \propto R^k$, where $k > 3$, then energy dissipation may be reduced by going to a plurality of smaller cavities to contain a given mass of liquid. The experimental determination that k lies between 4.10 and 5.45, supports a design decision to use a plurality of smaller cavities.

However, the greatest advantage is obtained, not due to the mere dependence of P on some power of R, but due to the effect of the parameter ζ. Fully turbulent flow is associated with the region $\zeta \leq 1$ and large energy dissipation rates. Very slow laminar flow is associated with the region $\zeta \geq 80$ and small energy dissipation rates. The reduction of R to a value that places ζ in the region above 80 should be the desired objective. Note that in the laminar case, the dependence of P on $R^{5.25}$ to $R^{5.45}$ leads to the further conclusion that even after laminar flow is reached additional reductions in P can be achieved by further reducing the tank size.

The introduction into the cavity of some type of fine mesh baffle is indicated if energy dissipation is to be minimized. Indeed, a preliminary test incorporating a coarse baffle in the 22-cm-diam sphere filled with water shows the baffle to be extremely effective in reducing energy dissipation. The use of some system of baffles would permit one or a few large tanks of any convenient shape.

References

[1] Vanyo, J. P. and Likins, P. W., "Measurement of Energy Dissipation in a Liquid-Filled, Precessing, Spherical Cavity," *Journal of Applied Mechanics*, Vol. 38, Ser. E, No. 3, Sept. 1971, pp. 674–682.

[2] Vanyo, J. P. and Likins, P. W., "Rigid-Body Approximations to Turbulent Motion in a Liquid-Filled, Precessing, Spherical Cavity," *Journal of Applied Mechanics*, Vol. 39, Ser. E, No. 1, March 1972, pp. 18–24.

[3] Vanyo, J. P., "An Energy Assessment for Liquids in a Filled, Precessing, Spherical Cavity," *Journal of Applied Mechanics*, to be published.